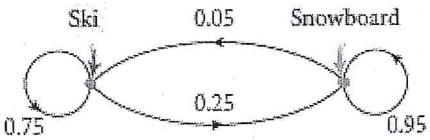
LESSON

6.1 Matrix Representations

Today \ Tomorrow	Ski	Snowboard
Ski	0.75	0.25
Snowboard	0.05	0.95

This entry shows that 5% of people who snowboard today will ski tomorrow.



OBJECTIVES

- Create transition diagrams and transition matrices
- Relate transition diagrams to matrices
- Understand matrices as a way to organize information
- Learn vocabulary associated with matrices: *dimension, row, column, entry*
- Find subsequent totals from the given initial value and percentages

On Saturday, Karina surveyed visitors to Snow Mountain with weekend passes and found that 75% of skiers planned to ski again the next day and 25% planned to snowboard. Of the snowboarders, 95% planned to snowboard the next day and 5% planned to ski. In order to display the information, she made this diagram.

Diagrams like these are called **transition diagrams** because they show how something changes from one time to the next. The same information is sometimes represented in a **transition matrix**. A **matrix** is a rectangular arrangement of numbers. For the Snow Mountain information, the transition matrix looks like this:

In mathematics, a **matrix** (plural matrices, or less commonly matrixes) is a rectangular array of numbers, symbols, or expressions.

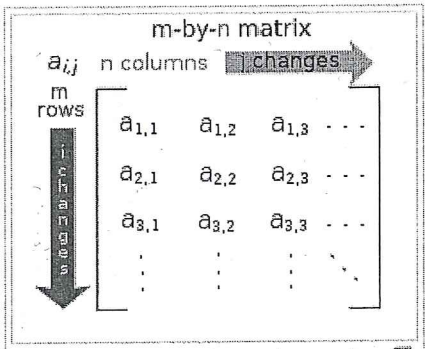
Rows go across like rowing a boat.

Columns go up or down like columns of a building.

Entries or Elements are the contents of a specific row and column.

Dimensions of a Matrix The number of rows and columns of a matrix, written in the form rows×columns. The matrix on the below has 2 rows and 3 columns, so its dimensions are 2×3. This is read aloud, "two by three."

$$\begin{bmatrix} 2 & 6 & -5 \\ 3 & 1 & 0 \end{bmatrix}$$

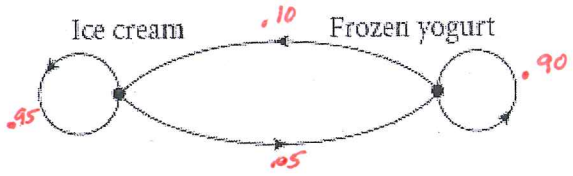


Specific elements of a matrix are often denoted by a variable with two subscripts. For instance, $a_{2,1}$ represents the element at the second row and first column of a matrix **A**.

Investigation • Chilly Choices

The school cafeteria offers a choice of ice cream or frozen yogurt for dessert once a week. During the first week of school, 220 students choose ice cream but only 20 choose frozen yogurt. During each of the following weeks, 10% of the frozen-yogurt eaters switch to ice cream and 5% of the ice-cream eaters switch to frozen yogurt.

Step 1 Complete a transition diagram that displays this information.



Step 2 Complete a transition matrix that represents this information. The rows should indicate the present condition, and the columns should indicate the next condition after the transition.

Next week

This week	Ice cream	Yogurt
Ice cream	.95	.05
Yogurt	.10	.90

Step 3 In the second week, how many students choose ice cream and how many students choose frozen yogurt? IC=220 FY=20

ice cream :	frozen yogurt
.95 (220) = 209	.05 (220) = 11
+ .10 (20) = 2	.90 (20) = 18
211	29

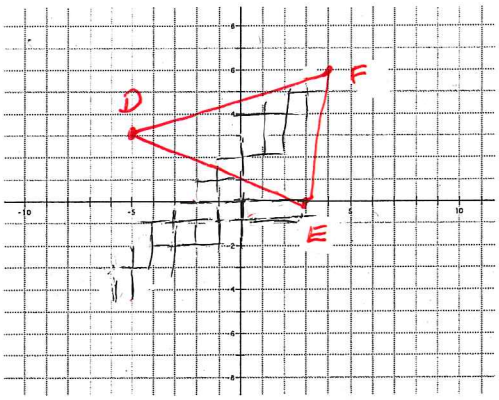
Step 4 How many students will choose each option in the third week?

ice cream	frozen yogurt
.95 (211) = 200.45	.05 (211) = 10.55
.10 (29) 2.9	.90 (29) = 26.1
203.35	36.65
≈ 203	≈ 37

3. Matrix M represents the vertices of $\triangle DEF$:

$$[M]=\begin{bmatrix} -5 & 3 & 4 \\ 3 & 0 & -6 \end{bmatrix}$$

a. Name the coordinates of the vertices and graph the triangle:



b. What matrix represents the image of after a translation $\triangle DEF$ left 3 unites?

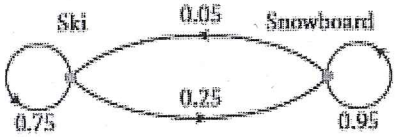
$$\begin{bmatrix} -8 & 0 & 1 \\ 3 & 0 & -6 \end{bmatrix}$$

c. What matrix represents the image of $\triangle DEF$ after a translation right 4 units and down 3 units?

$$\begin{bmatrix} -1 & 7 & 8 \\ 0 & -3 & -9 \end{bmatrix}$$

EXAMPLE B

In Karina's survey from the beginning of this lesson, she interviewed 260 skiers and 40 snowboarders. How many people will do each activity the next day if her transition predictions are correct?



$$\begin{bmatrix} 0.75 & 0.25 \\ 0.05 & 0.95 \end{bmatrix}$$

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Step 5 Write a recursive routine to take any week's values and give the next week's values.

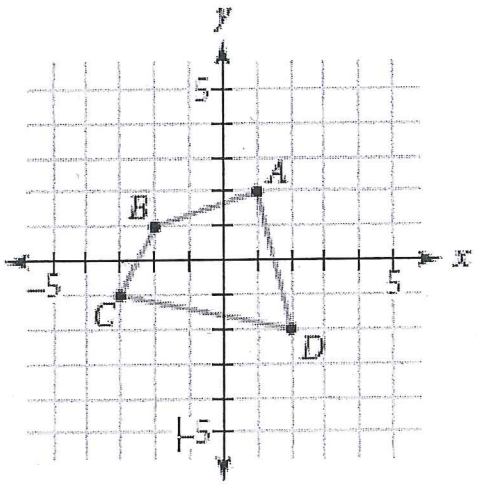
$f_1 = 20$ $\quad i_1 = 220$ $\quad n \geq 2$

$f_n = .05i_{n-1} + .90f_{n-1}$ $\quad i_n = .95i_{n-1} + .10f_{n-1}$

Step 6 What do you think will happen to the long-run values of the number of students who choose ice cream and the number who choose frozen yogurt?

$FY = 80$ $\quad IC = 160$

EXAMPLE A | Represent quadrilateral $ABCD$ as a matrix, $[M]$.



$$\begin{matrix} & A & B & C & D \\ x & 1 & -2 & -3 & 2 \\ y & 2 & 1 & -1 & -2 \end{matrix}$$

What matrix represents the image of ABCD after a translation down 2 units?

$$\begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & -3 & -4 \end{bmatrix}$$

What matrix represents the image of ABCD after a translation right 3 units?

$$\begin{bmatrix} 4 & 1 & 0 & 5 \\ 2 & 1 & -1 & -2 \end{bmatrix}$$